



# A new modelling approach for stabilisation of smart grids

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1. Problem statement and model description
2. Quadratic eigenvalue problem and clustering
3. Numerical experiments
4. Outlook



- ▶ Investigate simplified power system models, here the SP-Model [1]:

$$\begin{aligned} M_i \ddot{\theta}_i + D_i \dot{\theta}_i &= P_{M,i} - \sum_j B_{ij} V_i V_j \sin(\theta_i - \theta_j) & i \in \mathcal{V}_G \\ D_i \dot{\theta}_i &= P_{L,i} - \sum_j B_{ij} V_i V_j \sin(\theta_i - \theta_j) & i \in \mathcal{V}_L \end{aligned} \quad (1)$$

- ▶ Synchronized if  $\dot{\theta}_i = \dot{\theta}_j \forall i, j$  and the phase difference between connected nodes is bounded.
- ▶ Sophisticated conditions for existence of synchronization proposed in [2].

- ▶ Linearization of the full model around synchronized solution leads to:

$$\tilde{M}\ddot{\theta} + D\dot{\theta} = P - L\theta \quad (2)$$

- ▶ Can be solved by  $\theta = \sum_j \psi_j \exp(\lambda_j t)$ .
- ▶ Resulting in quadratic eigenvalue problem (QEP):

$$\left( \lambda^2 \tilde{M} + \lambda D + L \right) \psi = 0 \quad (3)$$

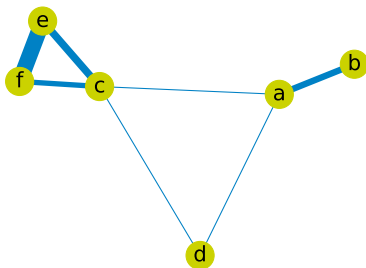
- ▶ To study the QEP, use a technique called linearization, leading to the companion form.
- ▶ Using  $u = \lambda v$ , we rewrite the QEP (3) as:

$$\lambda \underbrace{\begin{pmatrix} C & M \\ \mathbf{1} & \mathbf{0} \end{pmatrix}}_{:=A} \underbrace{\begin{pmatrix} v \\ u \end{pmatrix}}_{:=x} = \underbrace{\begin{pmatrix} -K & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}}_{:=B} \begin{pmatrix} v \\ u \end{pmatrix} \quad (4)$$

- ▶ Which is a GEP for the companion matrices  $(A, B)$ .

- ▶ Dynamical behavior can be found by eigenvalues and eigenvectors of the system.
- ▶ Nodes with identical eigenvector components for given eigenvalue will have identical behavior in this mode.
- ▶ Clustering can be observed, related to coherency of generators.
- ▶ For  $D = 0$ , studied in [3], main results:
  - ▶ Load nodes (where  $M_i = 0$ ) do not influence the dynamic directly and their behavior is only affected by the neighboring generator nodes.
  - ▶ Strongly connected nodes show slow coherency in the fastest modes (smallest eigenvalues).

# Example for clustering



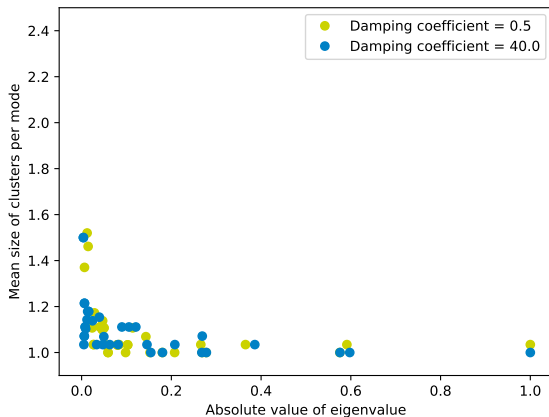
$\lambda$	0.000	0.109	0.232	1.309	1.932	3.618
a	0.408	-0.460	0.198	0.758	-0.092	0.003
b	0.408	-0.563	0.322	-0.641	0.042	-0.001
c	0.408	0.336	0.089	0.063	0.837	-0.085
d	0.408	-0.137	-0.898	-0.074	-0.043	0.002
e	0.408	0.410	0.143	-0.048	-0.299	0.744
f	0.408	0.414	0.147	-0.058	-0.444	-0.663

- ▶ Generalize results of [3], focus on clustering and coherency.
- ▶ Definition for cluster:
  - ▶ The eigenvector components corresponding to the nodes of  $C$  for a given mode are similar <sup>1</sup>.
  - ▶ The subgraph induced by  $C$  on  $G$  is connected.
- ▶ Investigate various topologies and parameter sets, in the following consider IEEE 30 test case.

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<sup>1</sup> $x, y$  are similar iff  $|x - y| \leq (10^{-1} + 10^{-4} |y|)$





**Figure:** Size of clusters with respect to mode for two different damping values.

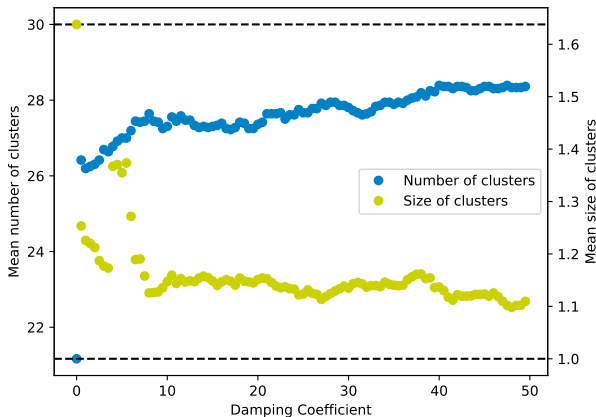
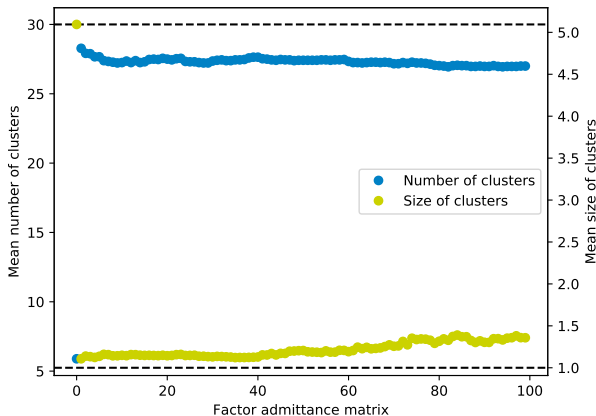


Figure: Number and size of clusters for different damping coefficients.



**Figure:** Number and size of clusters with respect to coupling strength of the network.

- ▶ Continuing investigating different IEEE and random graphs to assess how the parameters and topology influence clustering and dynamics.
- ▶ Investigate the nature and composition of clusters.
- ▶ Theoretical approach, based on [3], assess whether results are extendable by eigenvalue perturbation of original problem.



- ▶ Secondment at IREC: Try to incorporate simple control model in simplified power system model.
- ▶ Conditions to synchronize un-synchronized solutions.
- ▶ Influence of control on the stability of the synchronized solution, as studied here.



- [1] Takashi Nishikawa and Adilson E Motter. “Comparative Analysis of Existing Models for Power-Grid Synchronization”. In: *New Journal of Physics* 17.1 (Jan. 27, 2015), p. 015012.
- [2] F. Dorfler, M. Chertkov, and F. Bullo. “Synchronization in Complex Oscillator Networks and Smart Grids”. In: *Proceedings of the National Academy of Sciences* 110.6 (Feb. 5, 2013), pp. 2005–2010.
- [3] Babak Ayazifar. “Graph Spectra and Modal Dynamics of Oscillatory Networks”. Massachusetts Institute of Technology, 2002.

