# INCITE summer school 2017
# IRP32: A new modelling approach for stabilisation of smart grids

**Felix Koeth**

**G2Elab Grenoble**

June 30, 2017

**This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318**

# Outline



This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Power studies and synchronization

- Stable operation of the power system depends on the ability of the power system to synchronize
- Synchronization is also found in other physical systems, like biological, mechanical or chemical oscillators
- In simplified power system models, sophisticated synchronization conditions can be found.
- Main goal of my work: Expand the models and try to expand current synchronization conditions for this more complex models

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

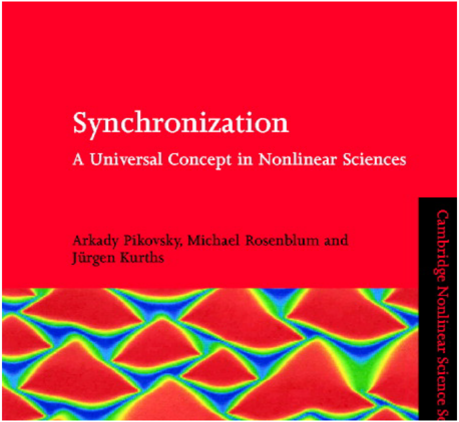
# Synchronization - I

- Synchronization in dynamical systems refers to a coordinate behavior
- All oscillators rotate with a common frequency
- Angular differences are bounded
- Corresponds to the equilibrium solution to dynamical models
- Stability of synchronized solutions if equilibrium is stable

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Synchronization - II

If anyone is interested in synchronization:




Or wait for my blog entry this month . . .

# Structure-preserving model and previous results

One of the most commonly simplified model to study the power system dynamics is the structure-preserving model:

$$M_i\ddot{\theta} + D_i\dot{\theta} = \omega_{i,\mathsf{p}} + \underbrace{\sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j)}_{P_{e,g}} \quad i \in \mathcal{V}_{\mathcal{G}} \tag{1}$$

$$D_i\dot{\theta} = \omega_{i,\mathsf{p}} + \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) \quad i \in \mathcal{V}_{\mathcal{L}} \tag{2}$$

For this model, the existence and stability of a synchronized solution is given if [1][1]:

$$\left\| L^{\dagger}\omega \right\|_{\varepsilon,\infty} \leq 1 \tag{3}$$

[1] With the (pseudo-inverse) network laplacian $L$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Reactive power

- The structure-preserving model neglects reactive power and voltage dynamics
- Especially for load buses, reactive power flow will have a serious impact on the voltage magnitude
- The voltage magnitude at each bus influences the coupling strength of the network, and thus the synchronization properties
- Modeling the reactive power and treating the (load bus) voltage magnitudes as variables results in a DAE system with the algebraic constraint:

$$Q_i = -\sum_j V_i V_j B_{ij} \cos(\theta_i - \theta_j) \tag{4}$$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Full model

- Proposed model: Model generators with the swing equation (as PV buses) and loads as constant power loads
- At load bus, voltage magnitude and phase are algebraic variables (as PQ buses)

$$M_i\ddot{\theta} + D_i\dot{\theta} = \omega_{i,\mathsf{p}} + \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) \quad i \in \mathcal{V}_{\mathcal{G}} \tag{5}$$

$$P_i = \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) \quad i \in \mathcal{V}_{\mathcal{L}} \tag{6}$$

$$Q_i = -\sum_j V_i V_j B_{ij} \cos(\theta_i - \theta_j) \tag{7}$$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Problems with DAE

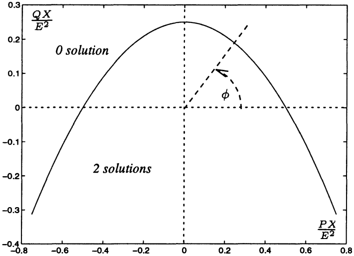
- Plans for the extended model
  - Numerical simulations
  - Finding equilibrium points
  - Calculate stability of equilibrium points
- For a solution, we need consistent initial conditions $\rightarrow$ equilibrium points
- Existence of solutions is not always guaranteed, even for consistent initial conditions

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Loadability condition

- In steady state, the full model corresponds to the full load-flow equations, which have to fulfill the loadability limit [2]
- The limit of the loadability is given by $y^*$, which can be found by finding:

$$\det J_{y^*} = \det \begin{pmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{pmatrix}_{y^*} = 0$$




This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Small signal stability

- Studying the behavior around an equilibrium point (where the loadability limit is valid) by linearization
- With the error variables $\tau_i = \theta_i - \theta_i^*$ and $\nu_i = V_i - V_i^*$.

$$\begin{pmatrix} M_i \ddot{\tau}_i + D_i \dot{\tau}_i \\ 0 \end{pmatrix} = J_{y^*} \begin{pmatrix} \tau \\ \nu \end{pmatrix} \tag{8}$$

Again, the jacobian matrix plays an important role in the dynamics of the full model.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Jacobian of load-flow equations

$$\frac{\partial P_i}{\partial \theta_j} = \begin{cases} \sum_j V_i^* V_j^* B_{ij} \cos(\theta_i^* - \theta_j^*), & \text{if } i = j \\ -V_i^* V_j^* B_{ij} \cos(\theta_i^* - \theta_j^*), & \text{if } i \neq j \end{cases}$$

$$\frac{\partial P_i}{\partial V_j} = \begin{cases} \sum_j V_j^* B_{ij} \sin(\theta_i^* - \theta_j^*), & \text{if } i = j \\ V_i^* B_{ij} \sin(\theta_i^* - \theta_j^*), & \text{if } i \neq j \end{cases}$$

$$\frac{\partial Q_i}{\partial \theta_j} = \begin{cases} -\sum_j V_i^* V_j^* B_{ij} \sin(\theta_i^* - \theta_j^*), & \text{if } i = j \\ V_i^* V_j^* B_{ij} \sin(\theta_i^* - \theta_j^*), & \text{if } i \neq j \end{cases}$$

$$\frac{\partial Q_i}{\partial V_j} = \begin{cases} \sum_j V_i^* V_j^* B_{ij} \cos(\theta_i^* - \theta_j^*), & \text{if } i = j \\ V_i^* V_j^* B_{ij} \cos(\theta_i^* - \theta_j^*), & \text{if } i \neq j \end{cases}$$

With Laplacian and non-laplacian structure.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Laplacian structure

- Laplacian of graph is a common tool used in graph theory. Encodes the structure of the grid
- For a weighted graph with weights $a_{ij}$ between vertex $i$ and $j$, the laplacian matrix is given as:

$$L = \begin{pmatrix} \sum_j a_{0j} & -a_{01} & -a_{02} & \cdots & -a_{0n} \\ -a_{10} & \sum_j a_{1j} & -a_{12} & \cdots & -a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{n0} & -a_{n1} & -a_{n2} & \cdots & \sum_j a_{nj} \end{pmatrix} \tag{9}$$

- $L$ is a symmetric matrix (only real eigenvalues) with the non-degenerate eigenvalue 0[2]
- The laplacian is linked to the consensus protocol and the dynamics of the linearized load-flow equations for the active power.

[2] For neglected reactive power and constant voltage magnitudes, the system is always at the loadability limit.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Other submatrix type

- The other Jacobian submatrix has a related structure to the Laplacian matrix, with positive off-diagonal elements:

$$M = \begin{pmatrix} \sum_j a_{0j} & a_{01} & a_{02} & \cdots & a_{0n} \\ a_{10} & \sum_j a_{1j} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n0} & a_{n1} & a_{n2} & \cdots & \sum_j a_{nj} \end{pmatrix} \tag{10}$$

- For a symmetric matrix $A$, $M$ is also symmetric. It is weakly diagonally dominant. In contrast to the laplacian, it is not always singular, depending on the structure of the graph.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Singularity properties

Singularity of matrices: Matrix is non-invertible, the determinant is zero, the matrix has a zero eigenvalue Nonsingular matrices are invertible!

- Full graph: nonsingular
- Ring graph: singular if $n$ even, nonsingular if $n$ odd
- Random graphs:
  - Erdős-Rényi graph graph: Nonsingular for $N > 20$
  - Random geometric graph: Nonsingular
  - Watts-Strogats graph: Singular in $\approx 50\%$ of cases.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Outlook

- Analytical treatment: Possible, impossible? Are there good ways to compare the Laplacian matrices with the almost-laplacian matrices $M$?
- The small signal model may be solved in the spectral domain, as was done for the constant voltage case in [3]. For that, the matrix properties of the jacobian, and especially the submatrices have to be studied.
- Concentrating on numerical simulations? Instabilities observed in the simulation, need to simulate control?

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Thanks for your attention!

Any questions?

# Bibliography I


[1] F. Dorfler, M. Chertkov, and F. Bullo. "Synchronization in complex oscillator networks and smart grids". In: *Proceedings of the National Academy of Sciences* 110.6 (Feb. 5, 2013), pp. 2005–2010.

[2] T. van Cutsem and K. Vournas Costas. *Voltage Stability of Electric Power Systems*. Kluwer international series in engineering and computer science. Springer, 1998.

[3] Nicolás Rubido. *Energy Transmission and Synchronization in Complex Networks*. Springer Theses. Cham: Springer International Publishing, 2016.

[4] Savoie J. Dubeau F. "A remark on cyclic tridiagonal matrices". eng. In: *Applicationes Mathematicae* 21.2 (1991), pp. 253–256.


This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Singularity of $M$ - I

Percentage of singular matrices $M$, for different weighted random graphs models (Watts-Strogatz graph WSG, Erdős-Rényi graph ERG and random geometric graphs RGG) and different realizations. Here, $N$ is the number of nodes and $p$ is the rewiring probability/radius. The Watts-Strogats graph is initially connected to two nearest neighbors.

| $N$ | $p$ | WSG | ERG | RGG |
|---|---|---|---|---|
| 10 | 0.15 | 54.16 | 32.1 | – |
| 10 | 0.3 | 45.445 | 3.86 | 1.58 |
| 10 | 0.6 | 38.3 | 0.0 | 0.0 |
| 10 | 0.75 | 38.08 | 0.0 | 0.0 |
| 20 | 0.15 | 48.58 | 0.44 | – |
| 20 | 0.3 | 46.54 | 0.0 | 0.0 |
| 20 | 0.6 | 44.96 | 0.0 | 0.0 |
| 20 | 0.75 | 44.16 | 0.0 | 0.0 |

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Singularity of $M$ - II

| $N$ | $p$ | WSG | ERG | RGG |
|---|---|---|---|---|
| 30 | 0.15 | 47.94 | 0.0 | – |
| 30 | 0.3 | 48.19 | 0.0 | 0.0 |
| 30 | 0.6 | 45.28 | 0.0 | 0.0 |
| 30 | 0.75 | 44.58 | 0.0 | 0.0 |
| 60 | 0.15 | 49.88 | 0.0 | – |
| 60 | 0.3 | 48.28 | 0.0 | 0.0 |
| 60 | 0.6 | 46.7 | 0.0 | 0.0 |
| 60 | 0.75 | 47.62 | 0.0 | 0.0 |
| 120 | 0.15 | 50.96 | 0.0 | – |
| 120 | 0.3 | 49.6 | 0.0 | 0.0 |
| 120 | 0.6 | 47.36 | 0.0 | 0.0 |
| 120 | 0.75 | 47.72 | 0.0 | 0.0 |

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

For complete graphs with $N$ edges, $M_{F,N}$ is given by:

$$M_{F,N} = \begin{pmatrix} N-1 & 1 & 1 & \cdots & 1 \\ 1 & N-1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & N-1 \end{pmatrix}$$

Testing for linear dependency with $x_i$, $i \in [0, N]$. Subtracting column $i$ from $j$ in the resulting linear system results in:

$$(N-2)x_i - (N-2)x_j = 0$$
$$\forall i, j : x_i = x_j \rightarrow x_i = 0$$

So, all columns of $M$ are linear independent and $M_{F,N}$ is non-singular.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Singularity of $M$ - IV

Ring graph with matrix $M_{R,n}$: singular if $n$ even, not singular if $n$ odd!

$$M_{R,n} = \begin{pmatrix} 2 & 1 & 0 & \cdots & 1 \\ 1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 2 \end{pmatrix} \quad \text{define:} \quad K_n = \begin{pmatrix} 2 & 1 & 0 & \cdots & 0 \\ 1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 2 \end{pmatrix}$$

For the tridiagonal matrix $K_n$ we have that $\det K_n = n + 1$[3]. According to [4], we can calculate the determinant of the cyclic tridiagonal matrix as:

$$\det M_{R,n} = \det K_n + (-1)^{n+1} \left( \prod_{i=1}^{n} a_i + \prod_{i=1}^{n} c_i \right) - a_1 c_n \delta_{n-1} \sum_{k=1}^{n-1} \frac{1}{\delta_{k-1}\delta_k} \left( \prod_{i=1}^{n} a_i \right) \left( \prod_{i=1}^{n} c_i \right)$$

[3] Because $\det K_n = \det K_{n-1} - \det K_{n-2}$

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318

# Singularity of $M$ - V

Here, $a_i = c_i = 1$ and $\delta_k$ is the $k$-th leading principal minor of the matrix $K_n$. Removing the last $m$ rows and columns from $K_n$ results in the matrix $K_{n-m}$, thus $\delta_k = k + 1$. With[4],

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

we get:

$$\det M_{R,n} = n + 1 + 2 \cdot (-1)^{n+1} - (n-1) = \begin{cases} 0, & \text{if } n \text{ is even} \\ 4, & \text{if } n \text{ is odd} \end{cases}$$

Does this explain the results for the WSG? Is that actually useful? Other matrix properties?

[4] Might be well known, can be shown by simple mathematical induction.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under Marie Sklodowska-Curie grant agreement No 675318